UNIVERSITY OF MUMBAI
No. UG/96 of 2016-17

CIRCULAR:-

A reference is invited to the Syllabi relating to the B.A./B.Sc. degree course **vide** this office Circular No.UG/138 of 2011 dated 14th June, 2011and the Principals of the affiliated Colleges in Arts & Science are hereby informed that the recommendation made by Board of Studies in Mathematics at its meeting held on 15th June, 2016 has been accepted by the Academic Council at its meeting held on 24th June, 2016 **vide** item No. 4.51 and that in accordance therewith, the revised syllabus as per the Choice Based Credit System for F.Y.B.A/B.Sc. degree program in Mathematics (Sem.I & II), which is available on the University's web site (www.mu.ac.in) and that the same has been brought into force with effect from the academic year 2016-17.

24/10/16

MUMBAI – 400 032 (Dr.M.A.Khan)
25th October, 2016 REGISTRAR

To,

The Principals of the affiliated Colleges in Arts & Science.

A.C/4.51/24.06.2016

****

No. UG/96 -A of 2016 MUMBAI-400 032 25 October, 2016

Copy forwarded with Compliments for information to:-

1) The Deans, faculties of Arts & Science,
2) The Chairman, Board of Studies in Mathematics,
3) The Professor-cum-Director, Institute of Distance & Open Learning
4) The Director, Board of College and University Development,
5) The Co-Ordinator, University Computerization Centre,
6) The Controller of Examinations.

24/10/16

(Dr.M.A.Khan)
REGISTRAR

...PTO

UNIVERSITY OF MUMBAI
SYLLABUS for the F.Y.B.A/B.Sc.
Programme: B.A./B.Sc.
Subject: Mathematics

Choice Based Credit System (CBCS)
with effect from the
academic year 2016-17

Semester I

Calculus I				
Course Code	Unit	Topics	Credits	L/Week
USMT101,UAMT101	Unit I	Real Number system	3	3
	Unit II	Sequences		
	Unit III	Limits & Continuity		
Algebra I				
USMT102	Unit I	Integers & divisibility	3	3
	Unit II	Functions & Equivalence relation		
	Unit III	Polynomials		

Semester II

Calculus II				
Course Code	Unit	Topics	Credits	L/Week
USMT201,UAMT201	Unit I	Series	3	3
	Unit II	Continuous functions & Differentiation		
	Unit III	Applications of differentiation		
Linear Algebra				
USMT202	Unit I	System of Linear Equations & Matrices	3	3
	Unit II	Vector spaces		
	Unit III	Basis & Linear transformations		

Teaching Pattern

1. Three lectures per week per course. Each lecture is of 1 hour duration.
2. One tutorial per week per course (the batches to be formed as prescribed by the University)

Syllabus for Semester I & II

Note: All topics have to be covered with proof in details (unless mentioned otherwise) and with examples.

USMT101/UAMT101 CALCULUS I

Unit I: Real Number System (15 Lectures)
Real number system $\mathbb{R}$ and order properties of $\mathbb{R}$, Absolute value $|.|$ and its properties.

AM-GM inequality, Cauchy-Schwarz inequality, Intervals and neighbourhoods, Hausdorff property.

Bounded sets, statement of l.u.b. axiom, g.l.b. axiom and its consequences, Supremum and infimum, Maximum and minimum, Archimedean property and its applications, density of rationals.

Unit II: Sequences (15 Lectures)
Definition of a sequence and examples, Convergence of sequence, every convergent sequence is bounded, Limit of a convergent sequence and uniqueness of limit , Divergent sequences.

Convergence of standard sequences like

$$\left(\frac{1}{1+na}\right) \forall\, a>0, \quad (b^n)\ \forall\, 0<b<1, \quad (c^{\frac{1}{n}})\ \forall\, c>0, \quad \&\ (n^{\frac{1}{n}}),$$

algebra of convergent sequences, sandwich theorem, monotone sequences, monotone convergence theorem and consequences such as convergence of $(1+\frac{1}{n})^n)$.

Definition of subsequence, subsequence of a convergent sequence is convergent and converges to the same limit, definition of a Cauchy sequence, every convergent sequence is a Cauchy sequence and converse.

Unit III: Limits & Continuity (15 Lectures)
Brief review: Domain and range of a function, injective function, surjective function, bijective function , composite of two functions (when defined), Inverse of a bijective function.

Graphs of some standard functions such as $|x|, e^x, \log x, ax^2+bx+c, \frac{1}{x}$, $x^n (n \geq 3), \sin x, \cos x, \tan x, x\sin(\frac{1}{x}), x^2\sin(\frac{1}{x})$ over suitable intervals of $\mathbb{R}$.

Definition of Limit $\lim_{x \longrightarrow a} f(x)$ of a function $f(x)$, evaluation of limit of simple functions using the $\epsilon - \delta$ definition, uniqueness of limit if it exists, algebra of limits , limit of composite function, sandwich theorem, left-hand-limit $\lim_{x \longrightarrow a-} f(x)$, right-hand-limit $\lim_{x \longrightarrow a+} f(x)$, non-existence of limits, $\lim_{x \longrightarrow -\infty} f(x)$, $\lim_{x \longrightarrow \infty} f(x)$ and $\lim_{x \longrightarrow a} f(x) = \pm\infty$.

Continuous functions: Continuity of a real valued function on a set in terms of limits, examples, Continuity of a real valued function at end points of domain, Sequential continuity, Algebra of continuous functions, Discontinuous functions, examples of removable and essential discontinuity.

Reference Books:

1. R. R. Goldberg, Methods of Real Analysis, Oxford and IBH, 1964.
2. K.G. Binmore, Mathematical Analysis, Cambridge University Press, 1982.
3. R.G. Bartle- D.R. Sherbert, Introduction to Real Analysis, John Wiley & Sons, 1994.

Additional Reference Books

1. T. M. Apostol, Calculus Volume I, Wiley & Sons (Asia) Pte. Ltd.
2. Richard Courant-Fritz John, A Introduction to Calculus and Analysis, Volume I, Springer.
3. AjitKumar-S. Kumaresan, A Basic Course in Real Analysis, CRC Press, 2014.
4. James Stewart, Calculus, Third Edition, Brooks/cole Publishing Company, 1994.
5. Ghorpade, Sudhir R.- Limaye, Balmohan V., A Course in Calculus and Real Analysis, Springer International Ltd, 2000.

Tutorials for USMT101, UAMT101:

1) Application based examples of Archimedean property, intervals, neighbourhood. 2) Consequences of l.u.b. axiom, infimum and supremum of sets. 3) Calculating limits of sequences. 4) Cauchy sequences, monotone sequences. 5) Limit of a function and Sandwich theorem. 6) Continuous and discontinuous functions.

USMT102 ALGEBRA I

Prerequisites:

Set Theory: Set, subset, union and intersection of two sets, empty set, universal set, complement of a set, De Morgan's laws, Cartesian product of two sets, Relations, Permutations n_{P_r} and Combinations n_{C_r}.

Complex numbers: Addition and multiplication of complex numbers, modulus, amplitude and conjugate of a complex number.

Unit I: Integers & divisibility (15 Lectures)
Statements of well-ordering property of non-negative integers, Principle of finite induction (first and second) as a consequence of well-ordering property, Binomial theorem for non-negative exponents, Pascal Triangle.

Divisibility in integers, division algorithm, greatest common divisor (g.c.d.) and least common multiple (l.c.m.) of two integers, basic properties of g.c.d. such as existence and uniqueness of g.c.d. of integers a & b and that the g.c.d. can be expressed as $ma + nb$ for some $m, n \in \mathbb{Z}$, Euclidean algorithm, Primes, Euclid's lemma, Fundamental theorem of arithmetic, The set of primes is infinite.

Congruences, definition and elementary properties, Eulers φ function, Statements of Eulers theorem, Fermats theorem and Wilson theorem, Applications.

Unit II: Functions and Equivalence relations (15 Lectures)
Definition of function; domain, co-domain and range of a function; composite functions, examples, Direct image $f(A)$ and inverse image $f^{-1}(B)$ for a function f; Injective, surjective, bijective functions; Composite of injective, surjective, bijective functions when defined; invertible functions, bijective functions are invertible and conversely; examples of functions including constant, identity, projection, inclusion; Binary operation as a function, properties, examples.

Equivalence relation, Equivalence classes, properties such as two equivalences classes are either identical or disjoint, Definition of partition, every partition gives an equivalence relation and vice versa.
Congruence is an equivalence relation on $\mathbb{Z}$, Residue classes and partition of $\mathbb{Z}$, Addition modulo n, Multiplication modulo n, examples.

Unit III: Polynomials (15 Lectures)
Definition of a polynomial, polynomials over the field F where $F = \mathbb{Q}, \mathbb{R}$ or $\mathbb{C}$, Algebra of polynomials, degree of polynomial, basic properties,

Division algorithm in $F[X]$ (without proof), and g.c.d. of two polynomials and its basic properties (without proof), Euclidean algorithm (without proof), applications, Roots of a polynomial, relation between roots and coefficients, multiplicity of a root, Remainder theorem, Factor theorem,

A polynomial of degree over n has at most n roots, Complex roots of a polynomial in $\mathbb{R}[X]$ occur in conjugate pairs, Statement of Fundamental Theorem of Algebra, A polynomial of degree in $\mathbb{C}[X]$ has exactly n complex roots counted with multiplicity, A non constant polynomial in $\mathbb{R}[X]$ can be expressed as a product of linear and quadratic factors in $\mathbb{R}[X]$, necessary condition for a rational number $\frac{p}{q}$ to be a root of a polynomial with integer coefficients, simple consequences such as $\sqrt{p}$ is a irrational number where p is a prime number, roots of unity, sum of all the roots of unity.

Reference Books

1. David M. Burton, Elementary Number Theory, Seventh Edition, McGraw Hill Education (India) Private Ltd.

2. Norman L. Biggs, Discrete Mathematics, Revised Edition, Clarendon Press, Oxford 1989.

Additional Reference Books

1. I. Niven and S. Zuckerman, Introduction to the theory of numbers, Third Edition, Wiley Eastern, New Delhi, 1972.

2. G. Birkoff and S. Maclane, A Survey of Modern Algebra, Third Edition, MacMillan, New York, 1965.

3. N. S. Gopalkrishnan, University Algebra, Ne Age International Ltd, Reprint 2013.

4. I .N. Herstein, Topics in Algebra, John Wiley, 2006.

5. P. B. Bhattacharya S. K. Jain and S. R. Nagpaul, Basic Abstract Algebra, New Age International, 1994.

6. Kenneth Rosen, Discrete Mathematics and its applications, Mc-Graw Hill International Edition, Mathematics Series.

Tutorials:

1. Mathematical induction (The problems done in F.Y.J.C. may be avoided).
2. Division Algorithm and Euclidean algorithm in Z, primes and the Fundamental Theorem of Arithmetic. 3. Functions (direct image and inverse image), Injective, surjective, bijective functions, finding inverses of bijective functions. 4. Congruences and Eulers-function, Fermat's little theorem, Euler's theorem and Wilson's theorem. 5. Equivalence relation. 6. Factor Theorem, relation between roots and coefficients of polynomials, factorization and reciprocal polynomials.

SEMESTER II

USMT 201 CALCULUS II

Unit I: Series (15 Lectures)

Series $\sum_{n=1}^{\infty} a_n$ of real numbers, simple examples of series, Sequence of partial sums of a series, convergence of a series, convergent series, divergent series, Necessary condition: $\sum_{n=1}^{\infty} a_n$ converges $\Rightarrow a_n \longrightarrow 0$, but converse is not true, algebra of convergent series, Cauchy criterion, divergence of harmonic series, convergence of $\sum_{n=1}^{\infty} \frac{1}{n^p}$ $(p > 1)$, Comparison test, limit comparison test, alternating series, Leibnitz's theorem (alternating series test) and convergence of $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$, absolute convergence, conditional convergence, absolute convergence implies convergence but not conversely, Ratio test (without proof), root test (without proof), and examples.

Unit II: Limits & Continuity of functions (15 Lectures)

Definition of Limit $\lim_{x \longrightarrow a} f(x)$ of a function $f(x)$, evaluation of limit of simple functions using the $\epsilon - \delta$ definition, uniqueness of limit if it exists, algebra of limits , limit of composite function, sandwich theorem, left-hand-limit $\lim_{x \longrightarrow a-} f(x)$, right-hand-limit $\lim_{x \longrightarrow a+} f(x)$, non-existence of limits, $\lim_{x \longrightarrow -\infty} f(x)$, $\lim_{x \longrightarrow \infty} f(x)$ and $\lim_{x \longrightarrow a} f(x) = \infty$.

Continuous functions: Continuity of a real valued function on a set in terms of limits, examples, Continuity of a real valued function at end points of domain, Sequential continuity, Algebra of continuous functions, Discontinuous functions, examples of removable and essential discontinuity. Intermediate value theorem and its applications, Bolzano- Weierstrass theorem (statement only): A continuous function on a closed and bounded interval is bounded and attains its bounds.

Differentiation of real valued function of one variable: Definition of differentiation at a point of an open interval, examples of differentiable and non differentiable functions, differentiable functions are continuous but not conversely, algebra of differentiable functions.

Chain rule, Higher order derivatives, Leibnitz rule, Derivative of inverse functions, Implicit differentiation (only examples).

Unit III: Applications of differentiation (15 Lectures)
Definition of local maximum and local minimum, necessary condition, stationary points, second derivative test, examples, Graphing of functions using first and second derivatives, concave, convex functions, points of inflection.

Rolle's theorem, Lagrange's and Cauchy's mean value theorems, applications and examples, Monotone increasing and decreasing function, examples,

L-hospital rule without proof, examples of indeterminate forms, Taylor's theorem with Lagrange's form of remainder with proof, Taylor polynomial and applications.

Reference Books:

1. R. R. Goldberg, Methods of Real Analysis, Oxford and IBH, 1964.
2. James Stewart, Calculus, Third Edition, Brooks/cole Publishing Company,1994.
3. T. M. Apostol, Calculus Vol I, Wiley & Sons (Asia) Pte. Ltd.

Additional Reference Books:

1. Richard Courant-Fritz John, A Introduction to Calculus and Analysis, Volume I, Springer.
2. Ajit Kumar- S. Kumaresan, A Basic Course in Real Analysis, CRC Press, 2014.
4. Ghorpade, Sudhir R.- Limaye, Balmohan V., A Course in Calculus and Real Analysis, Springer International Ltd, 2000.
5. K.G. Binmore, Mathematical Analysis, Cambridge University Press, 1982.
6. G. B. Thomas, Calculus, 12th Edition, 2009.

Tutorials: 1. Calculating limit of series, Convergence tests. 2. Properties of continuous functions. 3. Differentiability, Higher order derivatives, Leibnitz theorem. 4. Mean value theorems and its applications. 5. Extreme values, increasing and decreasing functions. 6. Applications of Taylors theorem and Taylors polynomials.

USMT 202/ UAMT 201 LINEAR ALGEBRA

Prerequisites: Review of vectors in $\mathbb{R}^2, \mathbb{R}^3$ and as points, Addition and scalar multiplication of vectors in terms of co-ordinates, dot-product structure, Scalar triple product, Length (norm) of a vector.

Unit I: System of Linear equations and Matrices (15 Lectures)
Parametric equation of lines and planes, system of homogeneous and non-homogeneous linear equations, the solution of system of m homogeneous linear equations in n unknowns by elimination and their geometrical interpretation for $(n, m) = (1, 2), (1, 3), (2, 2), (2, 3), (3, 3)$; definition of $n-$tuples of real numbers, sum of two $n-$tuples and scalar multiple of an $n-$tuple.

Matrices with real entries; addition, scalar multiplication and multiplication of matrices; transpose of a matrix, types of matrices: zero matrix, identity matrix, scalar matrices, diagonal matrices, upper triangular matrices, lower triangular matrices, symmetric matrices, skew-symmetric matrices, Invertible matrices; identities such as $(AB)^t = B^t A^t, (AB)^{-1} = B^{-1}A^{-1}$.

System of linear equations in matrix form, elementary row operations, row echelon matrix, Gaussian elimination method, to deduce that the system of m homogeneous linear equations in n unknowns has a non-trivial solution if $m < n$.

Unit II: Vector spaces (15 Lectures)
Definition of a real vector space, examples such as $\mathbb{R}^n,\ \mathbb{R}[X],\ M_{m\times n}(\mathbb{R})$, space of all real valued functions on a non empty set.

Subspace: definition, examples: lines, planes passing through origin as subspaces of $\mathbb{R}^2, \mathbb{R}^3$ respectively; upper triangular matrices, diagonal matrices, symmetric matrices, skew-symmetric matrices as subspaces of $M_n(\mathbb{R})$ $(n = 2, 3)$; $P_n(X) = \{a_0 + a_1X + \cdots + a_nX^n | a_i \in \mathbb{R}\ \forall\ 0 \le i \le n\}$ as a subspace $\mathbb{R}[X]$, the space of all solutions of the system of m homogeneous linear equations in n unknowns as a subspace of $\mathbb{R}^n$.

Properties of a subspace such as necessary and sufficient condition for a non empty subset to be a subspace of a vector space, arbitrary intersection of subspaces of a vector space is a subspace, union of two subspaces is a subspace if and only if one is a subset of the other.

Finite linear combinations of vectors in a vector space; the linear span $L(S)$ of a non-empty subset S of a vector space, S is a generating set for $L(S)$, $L(S)$ is a vector subspace of V; linearly independent/linearly dependent subsets of a vector space, a subset $\{v_1, v_2, \cdots, v_k\}$ of a vector space is linearly dependent if and only if $\exists\ i \in \{1, 2, \cdots k\}$ such that v_i is a linear combination of the other vectors $v_j's$.

Unit III: Basis and Linear Transformations (15 Lectures)
Basis of a vector space, dimension of a vector space, maximal linearly independent subset of a vector space is a basis of a vector space, minimal generating set of a vector space is a basis of a vector space, any two basis of a vector space have the same number of elements, any set of n linearly independent vectors in an $n-$dimensional vector space is a basis, any collection of $n+1$ linearly independent vectors in an $n-$dimensional vector space is linearly dependent; if $W_1,\ W_2$ are two subspaces of a vector space V then $W_1 + W_2$ is a subspace of the vector space $V$of dimension $\text{dim}(W_1) + \text{dim}(W_1) - \text{dim}(W_1 \cap W_2))$, extending any basis of a subspace W of a vector space V to a basis of the vector space V.

Linear transformations; kernel kernel(T) of a linear transformation, matrix associated with a linear transformation, properties such as: for a linear transformation T kernel(T) is a subspace of the domain space of T and the image image(T) is a subspace of the co-domain space of T. If V, W are real vector spaces with $\{v_1, v_2, \cdots, v_n\}$ a basis of V and $\{w_1, w_2, \cdots, w_n\}$ any vectors in W then there exists a unique linear transformation $T : V \longrightarrow W$ such that $T(v_j) = w_j \forall\ 1 \leq j \leq n$, Rank nullity theorem (statement only) and examples.
Reference Books:

1. Serge Lang, Introduction to Linear Algebra, Second Edition, Springer.
2. S. Kumaresan, Linear Algebra, A Geometric Approach, Prentice Hall of India Pvt. Ltd, 2000.

Additional Reference Books:

1. M. Artin: Algebra, Prentice Hall of India Private Limited, 1991.
2. K. Hoffman and R. Kunze: Linear Algebra, Tata McGraw-Hill, New Delhi, 1971.
3. Gilbert Strang: Linear Algebra and its applications, International Student Edition.
3. L. Smith: Linear Algebra, Springer Verlag.
4. A. Ramachandra Rao and P. Bhima Sankaran: Linear Algebra, Tata McGraw-Hill, New Delhi.
5. T. Banchoff and J. Warmers: Linear Algebra through Geometry, Springer Verlag, New York, 1984.
6. Sheldon Axler: Linear Algebra done right, Springer Verlag, New York.
7. Klaus Janich: Linear Algebra.
8. Otto Bretcher: Linear Algebra with Applications, Pearson Education.
9. Gareth Williams: Linear Algebra with Applications.

Tutorials:
1) Solving homogeneous system of m equations in n unknowns by elimination for $(m, n) = (1, 2), (1, 3), (2, 2), 2, 3), (3, 3)$, row echelon form.
2) Solving system $Ax = b$ by Gauss elimination, Solutions of system of linear Equations.
3) Verifying whether given $(V, +, \cdot)$ is a vector space with respect to addition $+$ and scalar multiplication $\cdot$
4) Linear span of a non-empty subset of a vector space, determining whether a given subset of a vector space is a subspace. Showing the set of convergent real sequences is a subspace of the space of real sequences etc.
5. Finding basis of a vector space such as $P_3(X),\ M_3(\mathbb{R})$ etc. Verifying whether a set is a basis of a vector space. Extending basis of a subspace to a basis of a finite dimensional vector space.
6. Verifying whether a map $T : X \longrightarrow Y$ is a linear transformation, finding kernel of a linear transformation and matrix associated with a linear transformation, verifying the Rank Nullity theorem.

Scheme of Examination

There will be a Semester end external Theory examination of 100 marks for all the courses of Semester I & II.
1. Duration: The examinations shall be of 3 Hours duration.
2. Question Paper Pattern: There shall be FOUR questions. The first three questions shall be of 25 marks on each unit, and the fourth question shall be of 25 marks based on Unit I, II, & III .
3. All the questions shall be compulsory with internal choices within the questions. Including the choices, the marks for each question shall be 38-40.
4. Questions may be subdivided into sub questions as a, b, c, d & e, etc & the allocation of marks depends on the weightage of the topic.

⋆⋆⋆⋆⋆⋆⋆⋆⋆⋆⋆⋆